Birzeit University
Mathematics Department
Math 337

Exam2 **Second Semester 2021/2022**

Answer any 10 of the following

1) Show that S_3 is not isomorphic to Z_6

S_3 is not cyclic but Z_6 is cyclic

2) Show that a group G of order 11 is cyclic

By lagrange's theorem any none identity element is a generator

3) Show that any infinite cyclic group is isomorphic to Z.

notes

4) Find all elements of order 10 in $Z_{10} \oplus Z_4$

see notes

5) Find all elements of order 8 in $Z_{10} \oplus Z_4$

none

6) If G is abelian group of odd order. Show that $f : G \to G$ defined by $f(x) = x^2$ is a group isomorphism

f is well-defined since if $a = b$, then $a^2 = b^2$, so $f(a) = f(b)$

Let $a, b \in G$. Then $f(ab) = (ab)^2 = a^2b^2 = f(a)f(b)$

Let $a, b \in G$ such that $f(a) = f(b)$, then $= a^2 = b^2$, so $(ab^{-1})^2 = e$, and so $|ab^{-1}| \leq 2$, but the order of G is odd so $|ab^{-1}| = 1$ and so $ab^{-1} = e$, and so $a = b$, and so f is one to one

Finally, let $|G| = k$ be odd and so there exist $m, n \in Z$ such that $2m + nk = 1$, and let $y \in G$, so $y^{2m+nk} = y^{2m}$ and so let $x = y^m \in G$, and $f(x) = y$, so it is onto

7) List all group isomorphisms from Z_8 into Z_8

Let $f : Z_8 \to Z_8$ defined by $f(1) = a$, so $|a|$ divides $|1| = 8$, so $|a| = 1, 2, 4, 8$ and so $a = 0, 4, 2, 6, 1, 3, 5, 7$

8) List all group isomorphisms from Z_8 into $Z_4 \oplus Z_2$

Let $f : Z_8 \to Z_4 \oplus Z_2$ defined by $f(1) = (a, b)$, so $|(a, b)|$ divides $|1| = 8$, and $|(a, b)|$ divides $|Z_4 \oplus Z_2| = 8$, so $|(a, b)| = 1, 2, 4$ and so $(a, b) = (0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1$

9) List all group isomorphisms from $Z_4 \oplus Z_2$ into $Z_2 \oplus Z_4$

$f : Z_4 \oplus Z_2 \to Z_2 \oplus Z_4$ defined by $f(1, 0) = (a, b), f(0, 1) = (c, d)$, so $|(a, b)|$ divides $|(1, 0)| = 4$, $|(a, b)|$ divides $|Z_2 \oplus Z_4| = 8$, so $|(a, b)| = 1, 2, 4$ and so $(a, b) = (0, 0), (0, 1), (1, 0), (1, 1), (0, 2), (1, 2), (0, 3), (1, 3)$. Similarly $|(c, d)|$ divides $|(0, 1)| = 2$ and divides $|Z_2 \oplus Z_4| = 8$

$(c, d) = (0, 0), (1, 0), (0, 2), (1, 2)$

10) If G is an abelian group of order 14. Show that G has an element of order 7

G is either isomorphic to D_7 or to Z_{14}, and since G is abelian and D_7 is not abelian so G is isomorphic to Z_{14} so it is cyclic and by FTOCG G has an element of order 7

11) State and prove Lagrange's Theorem.

12) Prove that A_4 has no subgroup of order 6

See notes